

THE POTENTIAL AND CHALLENGES OF USING GEOGEBRA TO TEACH GEOMETRICAL CONSTRUCTIONS IN BOTSWANA JUNIOR SECONDARY SCHOOLS: THE CASE OF GABORONE WEST GOVERNMENT SCHOOLS

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Abstract

This study discusses the potential and challenges of integrating technology in mathematics instruction using GeoGebra. There is a dearth of studies on the use of GeoGebra in classroom settings in the African context and the study was an effort to add to the literature. A case study method to gauge teachers' experiences on using the software to teach geometrical constructions was employed. The findings indicated that GeoGebra has the potential to support teachers if utilised effectively, but had its challenges. For example, teachers' lack of knowledge on ways to integrate ICT in the curriculum, how to exploit its features in mathematical activities and not enough or limited access to computer hardware were some of the obstacles and challenges perceived by teachers related to their inexperience with the software. The premise was that a teaching/learning environment based on using technology brings with it the possibility to work on mathematical concepts in a broader way compared to purely teacher-centred classroom sessions. Overall the study provide reasons to believe that GeoGebra has a potential in enhancing teachers' mathematics instructional practices as evidenced by the participants' general acceptance of its use in their lessons.

Keywords: *GeoGebra, teacher training, mathematics instruction, learner-centred, Botswana education*

Introduction

The study is based on the policy shift from the traditional instruction model of knowledge transmitting towards autonomous, active, and collaborative learning through students' engagement in information and communication technology (ICT)-based learning environments and shared learning resources as articulated in, RNPE, ETSSP, NDP 11, SDGs, P21 Framework, NCAF, OBE, BQA policy documents.

This study investigated the potentials and challenges of using one facet of ITC, the Dynamic Geometry System (DGS) software "GeoGebra" in mathematics instruction. This is in the backdrop of persistent decline in mathematics results attributed to teacher-centred pedagogy; inadequate and inappropriate use of resources; lack of interest and negative attitudes; failure to link mathematics to real life; the absence of problem-solving, discovery and investigation activities, and the incompetence of teachers in mathematical pedagogical content knowledge (PCK) (Taole & Chakalisa, 1995).

Innovations such as mixed-ability (differentiated) and remedial teaching have been employed to address students' underperformance, but to no avail (Republic of Botswana, 1994). In 2005 Botswana joined the Strengthening of Mathematics and Science in Secondary Education in Western, Eastern, Central and Southern Africa (SMASSE-WECSA), an in-service association of mathematics and science educators in twenty-nine (29) Sub-Sahara African countries established to motivate both teachers and students.

In the aftermath of joining SMASSE-WECSA, government recommended Microsoft Excel in Junior Certificate (JCE) mathematics syllabus and the authors believe that using only MS Excel in the syllabus tended to limit the development and improvement of the teaching and learning process.

The above developmental changes were as a result of the realisation that many students considered mathematics as rather boring, abstract, consisting of rules, laws, formulas and algorithms which they neither understand nor link to identifiable real-life scenarios. The absolutist teaching approach is a cultural norm in Botswana and teachers teach procedures which are forgotten within a short time span, hence this study advances a paradigm shift from abstract to an activity-based teaching; from a teacher-centred to a learner-centred methodology through the use of new technologies.

Such technologies are varied and differ, and GeoGebra was considered here for its availability as an open source application as well as for its user-friendliness. This study motivation was to interrogate the potentials and challenges encountered by mathematics teachers while using GeoGebra, bearing in mind that Botswana education aspires to produce globally technologically competitive citizens. Botswana aims to nurture and develop the creative elements within society, and actively contribute to the scientific and technological civilisation of the future (Vision 2016 towards Prosperity for All, 2004).

The traditional mode of teaching constructions, based on static drawings on paper where students then internalise abstract images and struggle to connect them to the theory on which such concepts are based to gain the knowledge being taught is no longer indispensable. The emergence of "new technologies have raised expectations about their potential for innovating teaching and learning" (Mariotti, 2013; p. 441). This has enabled teachers to exploit the potential of didactic software such as DGS to enhance mathematics learning and support teachers' classroom practices to overcome crucial difficulties in moving from an intuitive to a deductive approach to geometry (Mariotti, 2013).

Students taught with technology are more interested in learning mathematics and show improved performance (NCTM, 2010) as technology makes life easier for students, motivates them to learn and arouses interest in mathematics (Ofsted, 2008). This study assumed an innovative teaching strategy using GeoGebra that could motivate teachers to teach better and students to play an active role in learning mathematics.

Despite the fact that technology use has been neglected in Botswana for a very long time, partly because of teachers' incompetence in using it, this scenario is changing. For example, the Government of Botswana resolved sometime back in 1992 to provide computers to all public schools with the aim to improve performance. This has not however borne any fruits as students continue to perform poorly in mathematics (BEC 2011; TIMSS 2007). GeoGebra has been recognized as one of the first ICT tools being explored to achieve the integration of ICTs in education (NCTM, 2010), hence the exploration of its instructional use in Botswana herein.

Significance of the Study

The study investigated the potential use of GeoGebra in teaching geometrical constructions and the challenges thereof. The study findings would help curriculum developers and Ministry of Basic Education (MoBE) make informed decisions pertaining to mathematics instruction, the Botswana Examination Council (BEC) to implement alternative assessing methods with special reference to technology use and teacher educators and other stakeholders to adequately prepare teachers to use technology. Parents could make wise decisions on the choice of technological devices and software to acquire for assisting their children's learning.

Research Questions

- What are the potentials of GeoGebra in assisting teachers teach geometrical constructions?
- What is the acceptance level of teachers for the use of GeoGebra in teaching geometry?
- What challenges do teachers encounter when using GeoGebra in instructional processes?

Theoretical Framework

The constructivist paradigm and discovery learning theory were used to highlight and structure the range of approaches and methods to investigate the research questions. It is believed that knowledge is constructed by individuals based on a unique set of experiences with their environment (Crotty, 1998). The study did not therefore; assume any epistemological independence between social constructions and discovery learning, but rather that there are epistemological and ontological dynamics that create dialectic functionality between constructivism and discovery learning. While there are deep rooted differences between these views, their affinity to the use of technologies to enhance instructional practices is unquestionable.

With the help of technology, epistemological gains result from creating, constructing, discovering and negotiating rather than being told or given. Discovery learning theorists believe that humans acquire meaning through engaging and interpreting the world, and hence offering individuals the desire to know, which motivates them to solve problems (Surif, 2002). The theory challenges high achievers through open-ended problems mostly not offered in regular classroom sessions, while average achievers learn difficult concepts in a kid-friendly way by experimenting and comparing the results of their work.

Surif (2002) suggested three epistemological concepts from Bruner's theory namely: *enactive*-learning through action and manipulation; *iconic*- learning through the formation of images and organization of learning, seeing and kinaesthetic perceptions; and *symbolic*-learning through words and symbols. These evoke active participation to ensure clear understanding of concepts rather than rote learning (Omar, 2009) and provides experiences that help learners discover the underlying ideas, concepts or patterns.

Castronova (2009) condemned traditional instructional methods for not creating the employee for today's businesses. Technology availability requires new research to consider the effectiveness of technology-based discovery learning compared to using technology through traditional approaches. This calls for instructional models and strategies that focus on active, hands-on learning opportunities where students explore and solve

problems to create; integrate and generalize knowledge through student-driven, interest-based activities that encourage integration of new knowledge into existing knowledge (Holmes & Hoffman, 2000).

This encourages risk-taking, problem-solving and an examination of the learners' unique experiences as students drive the learning and are encouraged to learn at their own pace without being pushed, and this degree of flexibility evokes the zeal to learn without being inhibited by teachers. Using GeoGebra in teaching geometrical constructions allows students and teachers to construct meanings and discover new knowledge; hence the constructivist and discovery learning paradigms greatly informed this study.

The researchers went through a long protocol to request for the installation of GeoGebra software in school computers. Teachers had to rely on the internet to learn using the software before pedagogically incorporating it due to their low level of technological pedagogical content knowledge (TPCK).

The role of technology in education

Hawkrige (2009) outlined four principal rationales for introducing ICTs in instructional processes. *The Social Rationale* made it imperative for all students to become familiar with the computer and accept it in everyday use because of computer pervasion in societies globally. *The Vocational Rationale* relates to the need for computer education giving a possibility of better access to the job market by providing skills vital for employment. *The Catalytic Rationale* saw the introduction of computers as improving the overall performance of schools with more emphasis on problem solving approaches and learner-centered lessons which inculcate positive attitudes towards the education system in general. *The Pedagogic Rationale* asserts that computers assist and enhance teaching and learning processes with the teachers guiding the learning process while the learners do most of the work.

The permeation of technologies into education saw mathematics education as one of the earlier fields to introduce technology as an assistant tool in classrooms (Sangwin, 2007). The dynamic and symbolic nature of computer environments provoke students to generalise, formalise and make links between their initiatives, notions of mathematics and some formal aspects of mathematical knowledge. Technology has forced educators to re-evaluate the mathematics that students need and determine the best methods for attaining good results and positive mathematical attitudes. Most students find it difficult to learn mathematics, struggle to learn it, hate it, and feel it is not applicable to real life. Technology is a great equalizer that brings high and low achievers to the same level (Campoy, 1992) as both levels of students are taken to heights unknown.

According to NCTM (2000), six principles assist and guide teachers in improving the content and delivery of mathematics instruction, namely; equity, curriculum, teaching, learning, assessment and technology. Technology allows students to focus less on computational aspects and more on applications and Rojano (2001) posits that when technology is used appropriately students can learn more mathematics at a deeper level. They gain the opportunity of owning the mathematics being taught and the chance to model and conceptualise ideas, generate multiple representations of solutions, get instant feedback and have the chance to solve problems without the inconvenience of using pencil and paper.

Technology offers enticing possibilities for new pedagogical approaches across the curriculum through cognitive, metacognitive and affective channels. Mathematics analysis tools like graphic calculators, geometer sketchpad, statistics programmes, spreadsheets etc. can assist students' problem-solving, support exploration of concepts, provide dynamically linked representation of ideas and encourage general metacognitive abilities such as planning and checking. Appropriately designed mathematical software can help students learn mathematics better and students cannot "hide" what they do not know (Clements, 2003). However the use of technology in mathematics instruction brings with it some challenges for teachers and they

must be ready for it (Clements, 2003). Teachers are neither comfortable with using technology nor with giving up control of the classroom and the students.

Currently many children use technology in school and at home, which may influence and motivate learning. Rojano (2008) states that once computational aspects of mathematics are removed, the real learning begins. Technology allows students the luxury to focus on the concepts being taught and removes the burden of the computation associated with mathematics. The power of technology makes it possible and necessary to re-examine what mathematics students should learn and how they can learn it.

With technology, students explore new ideas, manipulate and discover certain mathematical concepts by themselves; organize and analyze computed data efficiently and accurately; participate in hands-on activities to see changes in variables; have a better scope of concepts and choices of steps to follow. It has multiple representations which enhance ability to visualize, and empowers students in multimodal learning. It provides access to mathematics for students with different abilities and increased opportunities to develop mathematical modeling skills and the ability to solve realistic problems rather than being restricted to abstract questions with one correct answer (Hawkrige, 2009).

Technology is therefore helpful in mathematics instruction and teachers should maximize its incorporation to empower students to learn. Technological tools address students' learning needs in terms of learning style preferences, pace, motivation, allowing hands-on activities, cooperative learning and verbalisation of thinking as students work as individuals and as groups to construct knowledge (Giamatti, 1995).

Dynamic Geometry Systems (DGS) in education

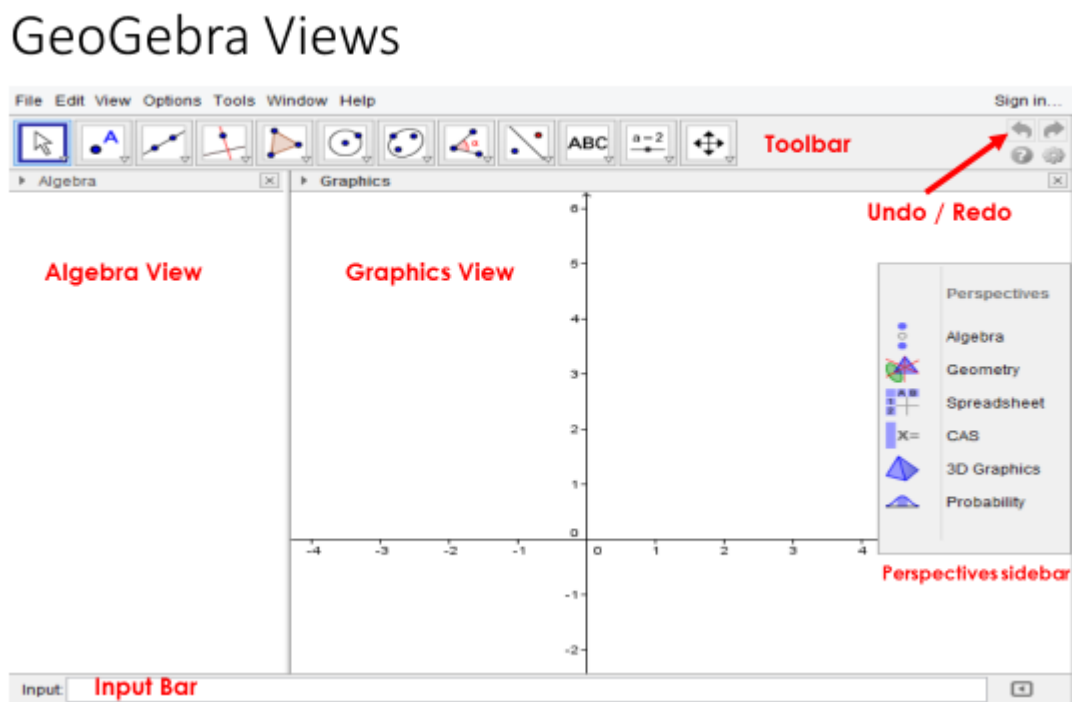
Research studies in the recent past have made reference to how students' behaviours changed from apprehension as they saw dragging on GDS as some form of distraction and interference due to being unfamiliar with seeing geometrical objects moving on paper, to excitement and contentment after experimenting and beginning to understand the power of the drag mode (Jones, 2002a). This indicates the conceptual potential offered by the DGS drag mode whose judicious use has been found to foster an understanding of proof. Nonetheless, research studies contend that DGS does not provide a self-contained environment, but requires other activities to help students make progress in mathematics. In the same vein, research shows that when DGS is used for conceptual exploration, it can lead to conceptual gain through the facilitation of some types of learning activities (exploration and visualisation) and the enhancement of others like proof and proving.

Within this understanding of the use of DGS arose the idea of semiotic mediation which Vygotsky (1978) referred to as "the potentiality that the use of a specific artefact has in fostering mathematical learning" (cited in Mariotti, 2013; p. 442). The DGS environment which incorporates the semantic domain of space and time can be interpreted according to this Vygotskian notion of semiotic mediation which alludes to the fact that through the process of internalisation, a technical tool may become a psychological tool and shape new meanings and therefore function as a semeiotic mediator, that is, an instrument of learning and conceptual understanding.

In that sense, DGS such as Cabri, GeoGebra, and others may become instruments of semiotic mediation if used by the teacher in the concrete realisation of classroom activities to introduce pupils to mathematical ideas. According to Mariotti (2013; p. 442-443) "The teaching-learning process starts with activities proposing to the students tasks to be accomplished using a specific artefact. The phenomenon of meanings' emergence in relation to the use of the artefact is called *unfolding of the semiotic potential*."

GeoGebra was the first among other tools being explored to achieve the integration of ICTs in education (NCTM, 2010). It dynamically integrates Geometry, Algebra and Calculus with elements of Dynamic Geometry Software (DGS) and Computer Algebra Systems (CAS). This *open-source* software provides different views (Figure 1)

Figure 1: GeoGebra window views



GeoGebra, like Geometer SketchPad, is ‘a dynamic construction and exploration tool that adds a powerful dimension to the study of mathematics’ (Key Curriculum Press, 2007) and can be introduced into the mathematics classroom at many levels, from basic Geometry, Algebra, to Calculus.

It has the potential to clearly demonstrate the close connection between geometry and algebra and is becoming a recognized part of mathematical knowledge (Hohenwarter and Preiner, 2007; Lu, 2007). According to Ogwel (2009), GeoGebra helps in:

- *Higher order thinking*: The activities are all linked and indicate mathematical connectedness, helping students to proceed from known to unknown, enabling them to use prior knowledge to acquire new knowledge through analysis, synthesis and evaluation giving them the platform to exercise critical thinking.
- *New knowledge*: GeoGebra affords generality of concepts and reveal mathematical structures with efficiency, unlike paper and pencil scenarios and students can derive their own formulas through making generalizations.
- *Learning Needs*: GeoGebra has the potential to create learning needs, by stimulating situations where current knowledge is inadequate in solving a problem, enhancing discovery of mathematics, its processes or versatility of applications in real life, allowing students to own their learning without concepts being imposed on them and offering them a platform to want to continue learning through further investigations.
- *Collaboration*: Students gain competence and confidence through practice, explorations, review of examples and readiness to seek help and share with the GeoGebra community.

- *Mediation*: GeoGebra, does not directly communicate the mathematical concepts, but encourages students to reason the basis of their findings, helping them realize mathematics as a reality rather than an abstract subject.

GeoGebra allows students to discover, explore, and investigate on their own with the teacher as facilitator. The inconvenience of using tangible instruments like protractor, compass and others mostly unavailable, easier to make mistakes with and may contribute to untidiness is eliminated. It gives students the opportunity to search information and shifts learning by memorization and mechanical procedures to encourage active participation and promote conceptual understanding, discovery, problem solving and analytical skills

The GeoGebra Institute South Africa 2010 report acknowledges the growth of GeoGebra in universities and other institutions of learning with teachers using it in their classroom on a daily basis. Examples include University of Pretoria, University of the Witwatersrand, Nelson Mandela Metropolitan University and others. Over and above institutional usage, there is AIMSSEC mathematics teachers contact sessions, conferences and workshops on GeoGebra such as SAARMSTE, National Congress of the Association for Mathematics Education of South Africa (AMESA) and others.

Other African countries have joined the GeoGebra Institute family such as GeoGebra Institute of Niger at Abdou Mamouni University of Niamey in cooperation with University of Cantabria in Spain; GeoGebra in Zambia at Chalimbana University; the recently formed GeoGebra Institute of Botswana and others. These are some of the examples of how GeoGebra has grown as a global software for classroom usage.

Problems associated with GeoGebra

Introducing GeoGebra in classroom settings raises the concern of teachers' inexperience with the software use. Students with special needs, for example, the visually impaired and those with continuous shaking and handling problems, may not fully explore the software, resulting in incomplete tasks. Though it allows students to work at their own pace, it may hinder the teacher to complete the syllabus on time. According to Clements (2003) teachers must be ready for the big change if they are to use geometric software, since even those with experience are sometimes not comfortable and confident to use it effectively. Teachers need to be empowered to use technology confidently and competently through continuous professional development (CPD) to be able to effectively contribute to their instructional practices. This is on the backdrop of the fact that adolescents are mostly influenced and fascinated by new trends.

Study Methodology

The study research design was underpinned by the constructivism framework and utilised qualitative data collection methods based on a case study whose data collection tools were interviews and observations. The case study research approach helped the researchers to examine the particularity and complexity of using GeoGebra in mathematics instruction. The methods involved detailed in-depth data collection and recordings of cases from multiple information sources (Creswell, 2007).

There is rapid growth in technology usage in secondary schools worldwide, but little or no research on GeoGebra uses in Botswana to guide this investigation. However, the researchers assumed that using GeoGebra for mathematics instruction could be useful, and that through case study explorations, a deeper understanding on how to adopt it for mathematics teachers could be attained. Case studies can be exploratory, descriptive or explanatory (Yin, 2003), and the object of this study was exploratory with some descriptive and explanatory elements used where and when necessary.

The study population comprised of government junior secondary school mathematics teachers in Gaborone West public schools in Botswana. Out of six junior secondary schools in Gaborone West, one school with 18 streams was conveniently selected for easy access as one of the researchers was based at the same school.

Purposive sampling was employed to select two female and two male mathematics teachers for observation teaching “*Geometrical constructions*” to their respective Form One classes. The teachers were selected due to their teaching experiences as well as their relative usage of technology for teaching and learning.

Mackenzie and Knipe (2006) advise that the paradigm underpinning the methodology and research questions should determine the particular data collection tools and analysis methods. Following government’s requirements on research protocol, permission to conduct research was sought and obtained from relevant authorities. Data was collected during second term of 2018, mainly through interviews and observations during class sessions with field notes taken to maximise information gathering techniques and to provide insightful and targeted evidence directly on the case study topic. The interview items were adapted from the framework proposed by Kaiser (2005) focusing on teachers’ perspectives on the use of technology. Data collected through interviews were documented as teachers shared their views on GeoGebra and the technical problems experienced while using the software.

The emerging issues were teachers’ conceptions of teaching geometry using GeoGebra, and in order to solicit more information, the researchers explored the reasons for, and ways of utilising GeoGebra, following a qualitative case study research approach methodology. Furthermore, informal interviews were conducted and field notes taken to get feedback on participants’ feelings as well as to obtain a more holistic sense of the way they used GeoGebra.

Validity and Reliability of Instruments.

The theoretical perspective of the researcher and the nature of reliability and validity are relative. In qualitative research, the validity of interviews is dependent upon depth, honesty, tenacity, veracity and objectivity of the researcher, while reliability refers to the extent to which research findings can be replicated (Merriam, 1998; Fraenkel & Wallen, 2003).

To ensure reasonable validity and reliability, the researchers recorded all conversations, allowing their contributions to be identified and enabling more careful analysis of participants’ responses to be carried out. This helped in reducing data distortion due to selective memory, thereby improving reliability of the study. Similar wording of open-ended questions were used for every participant in order to improve reliability of the interviews.

A pilot study was conducted on non-participating teachers to check both validity and reliability and their comments and views were used for further clarifications and explanations on the instruments. The researchers ensured proper alignment and coordination of the theoretical framework, research questions, research methodology, instruments and methods of data collection for this study.

Ethical Considerations

It was important to observe participants’ ethical perspectives; societal ethics, protocol and authority in position when carrying out the research study. The study focused on education in Botswana, and therefore permission was sought from the department of research and evaluation in the MoBE. The school-head of the

research site; the relevant senior teacher (senior teacher sciences) and the research participants were properly contacted. The contents of the interviews were treated in stringent confidentiality.

Data Analysis Procedures.

The data was analysed using a framework adapted from Kaiser (2005) focusing on teachers' perspectives on technology use to ensure comparability. This helped the researchers to explore and describe individual cases and get more insightful information. The following predetermined themes were used as a framework for the analysis of the transcripts.

Theme 1: Participants' Teaching Background: -qualification, years of teaching and experience using technology.

Theme 2: Teachers' Conceptions of GeoGebra: - conceptions of technology and GeoGebra in teaching and learning Geometry.

Theme 3: Software Evaluation: - strengths/weaknesses of GeoGebra compared to other software.

Theme 4: GeoGebra Usage: - ways of using GeoGebra, supporting materials and reasons for the chosen topics.

These themes gave proper direction to the data analysis protocol by helping the researchers identify and categorise related data for coherent presentation and interpretation. The collected data was coded into related sub-themes in accordance with the meaning derived thereof and allocated to the predetermined themes for interpretation.

Presentation and Discussion of Findings

To systematically report the cases, the researchers followed Thompson's (1984) framework for data analysis to discuss each case pertaining to the four themes using pseudonyms of the participants for confidentiality.

THE CASES

Case 1: Kemoeng

Qualification, Teaching Experience and Technology Usage.

Kemoeng has studied computer studies, is skilled in computer programming and has three years teaching experience. He teaches Form 1 to 3 with great interest in incorporating technology in mathematics instruction. This inadvertently gives Kemoeng an advantage over others with respect to the use of technology.

Conceptualisation of GeoGebra.

Kemoeng views GeoGebra as a tool that helps explore geometry rather than algebra. His experience with GeoGebra was just about a year, and felt underdeveloped in his capacity to use it. He had been very active in taking his students to the computer laboratory during mathematics lessons and they really liked using the dynamic software. However, he laments that students still have to master drawing using pencils as they are examined on this perspective rather than on using technology. He points out that GeoGebra, just like any tool can either be used badly or for the best, depending on different aspects.

There were no observed behavioural constructs suggesting a conflict between his beliefs about teaching with GeoGebra and his actual instructional practices, but rather a positive correlation between the two.

GeoGebra in relation to other software.

Kemoeng believes that GeoGebra is good for teaching geometry as a whole, and lamented; *“Though I have done a course on computer studies, I feel I still need to be resourced on how to effectively use GeoGebra to teach mathematics so that I can be confident with it and really master it. This will help my GeoGebra applied lessons to be more effective”*.

Using Technology in Teaching.

GeoGebra enabled his students to visualize parallel lines, make angle bisectors as well draw and measure different angles, and realised the linkage between properties of shapes, angles and their characteristics. He thinks the facilities for using this dynamic software are limited requiring improvements and. students should have more access to a fully equipped computer laboratory to benefit from it. Sharing of computers during lessons limited students to explore the software at their own pace.

Case 2:Lebogang

Qualification, Teaching Experience and Technology Usage.

Lebogang did computer studies as a minor course during her teacher training and taught mathematics for six years at junior secondary school level. The training had helped her to develop advanced skills in using mathematical software, such as Excel and GeoGebra which were already installed in the school computer laboratory.

Conceptualisation of GeoGebra.

Lebogang believes GeoGebra is a convenient tool in mathematics instruction which can arouse students' interest through visualisation, demonstrations and even checking ones' work. She stated; *“I like the software as it allows me to draw graphs, solve equations and even calculate. It has a lot of capabilities that other software like Excel do not have. I am happy about the software as it links geometry with algebra; it makes mathematics to be alive rather than the abstractness that is always perceived whenever the subject is mentioned.”*

She pointed out that both teachers and students do not view a computer as a learning tool but rather a tool for entertainment purposes, games or for communicating with friends over the internet. To her, the computer cannot do the logical and deductive thinking for both students and teachers and GeoGebra can only help to strengthen and motivate students to learn, but cannot explain why the concepts are right.

She laments at being discouraged by the current educational environment regarding the use of technology because computer laboratories are generally not well equipped for each student during class sessions as most of the computers are not working and students have to share. Students are prohibited from entering the computer laboratory without the teacher, which limits them to explore the dynamic software on their own. She saw GeoGebra as limited since human brains are the ones doing the logical thinking, but asserts that it provides quality functionalities that encourage her to use the software in teaching mathematics.

There were conflicts between Lebogang's rather negative views about technology and her classroom behaviour which endorses the use of GeoGebra as she was very involved in using the software during class observations. This suggests that some technology users have affinity in its use despite their misgivings about its general value in educational settings.

GeoGebra in relation to other software.

“I am rather impressed by the features and capabilities that GeoGebra has, moreover, there are other parts like tangent lines that can help students to explore more on geometry.”

Using Technology in Mathematics Teaching.

Lebogang used GeoGebra mostly in teaching geometry, demonstrations to emphasise key points, checking students’ work, testing, verifying thinking and sometimes for research with skilfulness in geometrical constructions.

Case 3: Tumiso

Background on Qualification, Teaching Experience and Technology Usage.

Tumiso had sixteen years of experience as a mathematics teacher, has taught Forms 1 to 3, acquired training in technology integration skills while doing her first degree at University of Botswana, gained particular interest and enthusiasm for new technologies, and likes trying a combination of open-source software to teach mathematics. She is head of mathematics and sciences department and encourages her subordinates to incorporate technology in teaching as stipulated in the recommendations of White Paper No. 2 of 1994.

Conception of GeoGebra.

Tumiso believes teachers are generally afraid of using computers in teaching mathematics, students are passive on using computers to learn as some fell asleep during demonstrations, suggesting they might not be interested in learning using the computer as they generally view it as a tool used for playing games, surfing the internet, chatting and others.

She views GeoGebra as very useful in rescuing one in times of difficulty instead of displaying and demonstrating on the chalkboard. It makes it easier for students to understand a particular situation when the point is dragged to see the effects of the motion of another point. She believes GeoGebra can be used to teach almost all mathematics topics, but for the software to benefit both students and teachers it must be used well, which calls for proper training. She laments that though the software is user-friendly, teaching mathematics using the software needs to be seriously addressed.

Observation of her sessions revealed that her beliefs about using technology and her actual behaviour in class were synchronous.

GeoGebra in relation to other software

Tumiso believes that GeoGebra has a great potential in terms of speed and its availability, performs better than other mathematical software that she knows. *“This software is very relevant to mathematics and I believe that if teachers can be trained on how to use it to teach mathematics, good results can be attained. I think that SMASSE, a programme for teaching mathematics can go hand in hand with the use of GeoGebra. Teachers may even be able to finish the syllabus as the software can help link the topics more easily”*, she said.

Using GeoGebra in Mathematics Teaching.

Tumiso used GeoGebra on geometrical topics and it changed her teaching strategies. She emphasises making step by step explanations for students to easily understand the concepts and commends the animation feature which attract students’ attention and appreciates the fact that GeoGebra is free for students and teachers to use at home, which can inculcate individual learning. She advocates for mathematics laboratories in schools and stresses that: *“Mathematics teachers should take students to the mathematics laboratory to investigate,*

explore and even manipulate objects to bring life to the subject and kill the abstractness that is generally associated with it. This may help bring better results and more mathematical orientated career aspirations by students at the end of the year.”

Case 4: Matthew

Qualification, Teaching Experience and Technology Usage.

Matthew had eight years teaching experience in different areas in Botswana making him understand students from different backgrounds. He has particular interest in using technology despite having no formal training on IT during teacher training. He has attended workshops on using Excel and is really into integrating technology in mathematics instruction.

Conception of GeoGebra

Matthew commends GeoGebra and believes it can arouse the interest of students to learn mathematics. *“It may be very useful for low and high achievers to see and explore more on the relationship between shapes, lines and angles. Students can really benefit from it and see the connection of mathematics topics as well as their relevance in real life”*. GeoGebra helped him bring life to mathematics and was excited about using the software; *“I believe that if I was well trained on using this software I could really make a great beneficial difference in mathematics classroom teaching. My students will also appreciate mathematics and be able to see the fascination and beauty of the subject”*.

Matthew’s classroom behaviour did not conflict his beliefs about teaching with technology. There was a positive relationship between his actions and teaching beliefs.

GeoGebra Evaluation in relation to other software.

Matthew claims that GeoGebra is more relevant than the recommended Excel as it covers geometrical topics, algebra and Excel itself and believes that government should encourage teachers to use it. He stressed that its use can kill the spirit of boredom that most students associate with mathematics.

Using Technology and GeoGebra in Mathematics Teaching.

Matthew mainly uses GeoGebra for demonstrations during his lessons and sometimes allows his students to explore and investigate concepts using the software. He also believes that though teachers are encouraged to incorporate technology in the teaching/learning of mathematics, they should not forget that students don’t use computers to write examinations but rather should be trained on how to use pencil and paper to learn and do constructions.

Cross-Case Analysis

Emerging Issues in Relation to the Use of GeoGebra

From the cases discussed above there are common issues among individual teachers which the researchers have extracted for cross-case analysis using a constant comparative method adopted from Glaser and Strauss (1967) and supported by a framework for analysing cross-cases from Dey (1993, pp.139) with a sequence of splitting, splicing categories and linking the data collected from the cases which helped in comparing the findings more effectively.

Most teachers mentioned the environment, the unavailability of facilities for students and teacher education on using technology to teach mathematics. They believe that if they had proper training on the use of GeoGebra, they could use the software for the greater benefit of the students. Most teachers view GeoGebra as an educational tool for demonstrations during class sessions and that there are certain mathematics topics that GeoGebra can address effectively.

The categories identified are: environment/infrastructure suitable for application of technology in mathematics teaching/learning, Teacher training on technology use for mathematics instruction; GeoGebra as a tool in teaching/learning mathematics and mathematics topics that GeoGebra can effectively be applied to enhance teaching and learning.

The Environment (Infrastructure and Resources).

Some of the respondents viewed GeoGebra as a friendly environment through which teachers and students can investigate, explore and manipulate objects to bring life to mathematics and kill the abstractness generally associated with it. They lament that computer laboratories are generally not well equipped for students' use during teaching and learning sessions, most computers are not working and students have to share during class sessions. Restricting students to enter the computer laboratory without the teacher, greatly limits them to explore the dynamic software.

There is no appropriate provision of materials to foster the use of technology. There are computer laboratories in all junior secondary schools throughout the country, but it is impractical for the many mathematics classes to be taught at the computer laboratory. Computer awareness lessons get the priority at the computer laboratory. This is a great disappointment to both teachers and students and teachers resort not to bother using the laboratory at all.

The findings suggest that mathematics teachers advocate for a conducive environment in terms of well-equipped laboratories which may foster GeoGebra use in teaching and learning, and would prefer a separate well equipped mathematics laboratory to allow students to investigate, explore and experiment their ideas to bring life to mathematics.

Mathematics Teacher Training on Technology

The study participants believe that it is one thing to use GeoGebra to learn and another to teach mathematics. They stressed the need to assimilate GeoGebra in the teaching/learning of mathematics on the basis that teachers be well trained on how to use the software for teaching. Since all the participants learnt GeoGebra through the internet, sharing ideas with each other and frequent use on their own, they felt they were not competent enough to thoroughly engage students during lessons, but mainly use it for demonstrations and presentations.

The findings suggest that teachers could be categorised into those with no formal training in computers in education and those with formal training. However, since the software is free, teachers have access to GeoGebra online materials from the internet for their classroom practices. With more time and experience teachers could be able to master their skills on the use of GeoGebra in teaching/learning mathematics.

Policies that govern a country have a great influence in curriculum development, and Botswana is no exception. The white paper No. 2 of 1994 (RNPE, 1994) recommended that technology should be infused in the teaching/learning of mathematics and the JCE mathematics syllabus was tailor made for teaching certain topics using technology.

The study revealed that most mathematics teachers have not acquired proper training for the incorporation of technology in instructional practices. GeoGebra has been received well by teachers and they were eager to learn the software on their own, showing their commitment and zeal to improve and develop their teaching through the technology. This suggests that, given formal in-service training on technology use, mathematics teachers can make a big difference in their instructional practices. Teacher educators should consider teaching pre-service mathematics teacher trainees on how to incorporate technology in mathematics instruction.

GeoGebra as an Educational Tool

In almost all the cases, teachers viewed GeoGebra as a relevant educational tool for both teachers and students and considered it as a tool to be used in classrooms for students to practice mathematics problem solving. A comparative analysis across the cases was applied and the findings are that GeoGebra is generally identified as an educational tool by mathematics teachers for classroom activities, investigations, demonstrations, presentations, visualisations, geometrical activities and proofs as well as for creating teaching materials, getting immediate feedback, reflective checking and for research purposes.

It can help speed up the process of preparing students for examinations. Students learn ahead without the hassle of having to use pen and pencil to draw shapes and graphs, visualize parallel lines, make angle bisectors as well draw and measure different angles and realise the linkage between properties of shapes, angles and their characteristics.

GeoGebra is revered as very useful for rescuing one in times of difficulty when displaying and demonstrating on the chalkboard, making it easier for students to understand mathematical concepts. It is good for mathematics explorations, investigations and manipulations and teachers advocated for mathematics laboratories in schools for students to applied mathematics practically and experience its beauty and relevance.

According to the interviews, teachers value the bidirectional capability of GeoGebra as a key feature which includes the drag mode and the inverse way of changeability in the algebraic window. Most participants valued GeoGebra as relevant for teaching mathematics compared to MS Excel.

The findings suggest that teacher conceptions appear to play a significant role in affecting instructional decisions and behaviour. For instance, most participants gave negative reflections about general technology integration, but were enthusiastic about using GeoGebra in mathematics instruction. Integration of technology in the teaching/learning of mathematics evokes many factors that interact with teachers' conceptions, decisions and behaviour. These could be their choices of mathematical software and pedagogical issues linking mathematical content knowledge and technology implementation.

Mathematics Topics addressed by GeoGebra

All the participants mainly used GeoGebra to teach geometrical constructions, and one lamented that students still had to master how to draw using pencils as they are examined on this perspective rather than on using technology. Participants felt that the software is very relevant to mathematics and if teachers could be trained to use it for teaching, good results could be attained.

The findings suggest that mathematics teachers have the willingness to further explore the potentials of GeoGebra to teach other topics than geometrical constructions. It is evident that if well trained, teachers will be competent enough to use it extensively in other areas of mathematics.

Conclusion

Geometrical constructions seem to form part of the most difficult concepts in the Botswana mathematics curriculum. As Schoenfield (1985) reported, the very nature of a construction problem makes it difficult to take a theoretical perspective and two dimensional static drawings can only make matters worse for most students. Battista (2007) attests to the fact that students find it hard to appreciate the difference between the actual (geometrical) figure (such as a kite) on paper and the theoretical object that it represents. According to Love (1995), geometry is the only area of mathematics where the physical geometrical image is both the actual object and its representation.

Given the difficulties evident in teaching/learning geometrical constructions, discovery learning provides a platform for teachers to allow students to determine the next step in the learning process, helps teachers to research continuously on their students, assess their students' needs and continue monitoring them to study how each student learn so they can cater for them. The theory guides both teachers and students towards using technology to help arouse students' interest as they are more inclined to be involved in their learning. As students discover, investigate, explore, discuss and experience, they become confident in mathematics and develop positive attitudes towards the subject, and realise its applications in real life situations.

According to Alagic (2003), teaching mathematics using technology requires teachers who can actually teach technology. Teachers tend to teach mathematics the way they have been taught as students and it is high time for a change to properly assimilate the global trends that are unavoidable in contemporary society. The government of Botswana recommended technology for incorporating in the teaching/learning of mathematics and it is the responsibility of mathematics teachers and professionals in the area to explore software that are relevant for implementation.

The findings show teachers' concerns interrelated within the issue of proper environment for technology use, training of teachers on the use of technology, understanding GeoGebra as an educational tool and the mathematics topics that can be effectively taught through technology. Given technology provision, teachers' mathematical content knowledge and conceptions may affect their pedagogical practices regarding utilising GeoGebra.

The study shows that GeoGebra could be used more than merely as a tool, but there is need for an environment where teachers and students collaborate for the creation of complete pieces of mathematical work. Implementation of GeoGebra in classroom practices can effectively result in sharing of mathematical ideas, thoughts, conjectures and investigations between teachers and students. GeoGebra as a free software is advantageous to both teachers and students, and teachers should try it even if they have never used technology before in their classroom practices. It is used worldwide there is great potential of instrumental features and the underlying mathematics within the use of GeoGebra as well as the pedagogical possibilities of open-source software into mathematics teaching/learning practices.

There were no suggestive indicators from the participants that DGS should replace traditional teaching methods, rather, the use of technology ushers in new strategies that serve to complement and reinforce knowledge gained from traditional teaching practices. This suggests a belief that the availability of DGS for geometrical constructions does not offer exclusive alternative approaches for its inclusion, but rather that constructions by hand should precede technology use. This means that learners need the traditional artefacts to move from intuition (theory) to practice and once this has been achieved, the use of DGS helps learners to better interpret and derive meanings from the concepts learnt. This is unavoidable in the developing world where technology use still has many challenges including inadequate resources, unreliable electricity supplies and teachers' lack of competence in technology use.

Recommendations

The researchers strongly believe that the pedagogy of GeoGebra should not be limited to presentations, as using it beyond demonstrations contributes to exploring challenges and potentials of its implementation. Therefore researchers recommend that the MoBE should recognise mathematical software like GeoGebra and train teachers through in-service training on how to teach mathematics using technology. Relevant software should be considered for particular subjects and be installed in schools for use by both teachers and students.

Pre-service teachers should also be trained well in the integration of technology in teaching mathematics so that they become well equipped to teach using technology. Internet services should be enhanced in schools to help teachers keep abreast with the use and application of technology in the teaching and learning of mathematics. This can also help share ideas on how to teach using technology in terms of chatting, e mails and others for both teachers and students.

To augment this, government should consider the provision of mathematics laboratories equipped with modern technology. There is evidence of the encroachment of smartboards in senior secondary school classrooms which is a positive development that could be extended to junior secondary schools. These smartboards could be used to complement other technological equipment such as computers and tablets that have found their way into the schools system.

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